Volume
1

HEATHCOTE INDUSTRIAL PLASTICS

Vibration & Vibration Damping

The Use of Composite Leaf Springs

in Vibrating Machinery

VIBRATION & VIBRATION DAMPING

A guide to the use of composite leaf springs in vibrating machinery

9 Silverdale Enterprise Park • Kents Lane
Newcastle-under-Lyme • Staffs • ST5 6SR
Phone (+44)(0)1782 444219• Fax (+44)(0)1782 610033
email info@heathcotes.com • web www.heathcotes.com

Table of Contents

Chapter 1

ICON KEY
Valuable information
Test your knowledge
Workbook review

Background

Like all things it's good to know a little bit about the history of the things that we use.

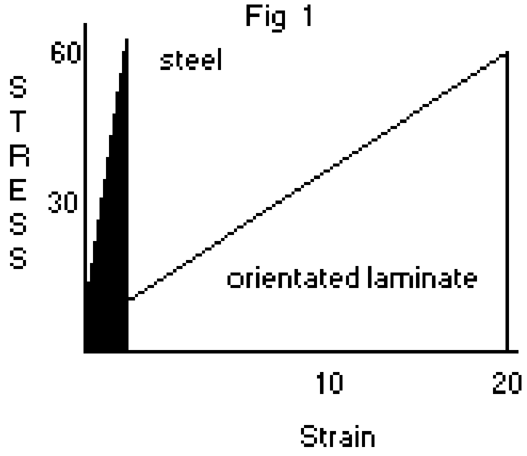
Flat leaf springs used in vibratory equipment are subjected to high bending forces in a single plane. In particular they must survive under very high "fatigue" conditions as they are expected to deflect many millions of times during their lifetime. It is common for a vibratory conveyor or feeder to deflect it's springs more than 4 million times in a day. To stand up to these forces and the smaller torsional forces that may develop because of misalignment or other reasons, a spring design was developed and patented in 1954 in which continuous glass filaments are specially orientated. In the surface and core layers (often referred to as "**plies**") the filaments are orientated longitudinally in the bending or primary direction but 10 to 15% of the filaments just under the surface are orientated at 90 degrees in order to give the required strength in the "cross" direction. This design is called "Spring configuration" in order to distinguish it from other orientations such as "unidirectional" and "crossply".

This special spring material also has a very high capacity for storing energy as illustrated in Figure 1 where it is compared to spring steel.

What does this graph mean?

It means that you can get more energy in to move the material .

Fig 1



Chapter 2

Equipment Considerations

There are a number of different types of vibratory equipment and before designing it is important to consider what general characteristics we need.

- Size is important but it shouldn’t be forgotten that this may often determine the type of drive that can best be used.

- Speed requirement is linked to size and this is generally the most common criteria upon which equipment is designed.

- The type of material being moved. The way that particles move is a big topic on it’s own but some basic principles can be followed. For this simple guide we will assume that the materials are basically free flowing and uniform.

- Conditioning of material may be important. If you want to move biscuits or crisps then you probably do not want to turn them into crumbs but if you are moving a “lumpy” product then you may wish to break up as many of the lumps as possible at the same time as you move the material. Alternatively your equipment may be a screen or sieve itself.

- Will the equipment be “direct drive” or “free mass drive”? Are the leaf springs helping with the actual drive or are they being used simply to give a flexible high fatigue support?

- Can we easily modify an existing design and what changes will be required? What are the practical limits of the modifications?

- What are the operational limits of the equipment required?

Chapter 3

Basic Formulae

We will not go into great depth here as this is not meant to be a text book but it is nice to know where the formulae that we use come from.

The first consideration is whether the equipment is designed to operate at it's "**Natural Frequency**" or not? All structures have a frequency that they feel "natural" at vibrating. An often used analogy is that of a children's swing. A small push at set regular intervals (or frequencies) keeps the swing going and the natural cycle of the swing returns it to it's starting point.

As a guide most very large equipment (quarrying , coal screens , paper making equipment etc.) are direct drive and rely totally on the drive to "push and pull" no matter how it would prefer to move. Here the dynamic force on the leaf springs is only dependent upon the drive, not on the weight. You should not use the natural frequency formula for this type of equipment but it can be helpful to let you know how far you are away from natural frequency. Composite leaf springs are used in direct drive systems predominantly for their fatigue life.

It is far more energy efficient to allow the natural flexing of the leaf springs to help the drive and this is the reason that this type of equipment has become so popular.

The natural frequency formula for leaf springs is derived from the angular velocity equation:-

$$\omega = \sqrt{\frac{K}{m}} \qquad \text{-------------------------------------}(1)$$

where ω = angular velocity = $2\pi F$

F = frequency , m = mass on spring , K = spring constant

converting equation (1) and solving for F and we get :-

$$F_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \qquad \text{--------------------------}(2)$$

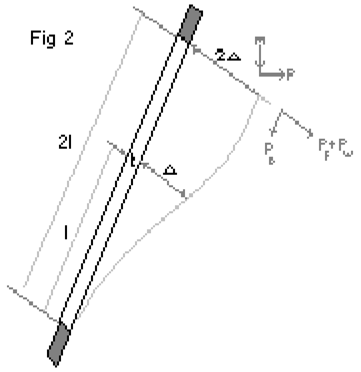
This formula is referred to as the "Natural frequency formula". If we set the natural frequency at the drive frequency of the equipment and know the mass on the springs then we can calculate the spring constant K.

With the frequency in Hertz and the mass in Kg then the spring constant is the force in Newton's needed to deflect the spring by 1mm if we use the following equation:-

$$K = \left(\frac{F_n}{5.03}\right)^2 \text{ x m}\text{-----------------------}(3)$$

This effectively tells us the stiffness of the spring that we require in order for it to vibrate at a natural frequency which is identical to the drive frequency of the equipment.

We next need to look at some beam theory in order to determine the dimensions of the leaf spring. The equation for the deflection of a cantilever beam is given by the following:-



$$\Delta = \frac{Pl^3}{3EI} \text{---}(4)$$

where P= force , I= moment of inertia of beam and E= Flexural Modulus

For a beam of rectangular cross section the moment of inertia is $1/12bt^3$. Usually the length and width of the spring have already been set by the design of the equipment and we are only trying to determine the thickness but this equation may be rearranged a number of different ways in order to tell us the possible dimensions of the spring design.

Substituting for the Inertia and rearranging for the thickness and we get:-

$$t^3 = \frac{4Pl^3}{Eb\Delta}$$ --------------------------------------(5)

where t= thickness , l= length and b= width

This equation allows us to determine the dimensions of the spring required.

Now we know what spring we can use but we also need to see how much stress is on the spring as we need to make sure that the springs will have a very long life.

One of the major benefits of well designed vibratory equipment is it's reliability. It is ideally suited to manufacturing processes that are intended to be operated continuously 24 hours/day , 7 days a week. The maintenance required by well designed vibratory equipment is minimal.

Chapter 4

Stress

If they last for 2 million cycles then there is a 98% probability of infinite fatigue life

The calculations in Chapter 3 do not tell us how long the springs will last?

After calculating the spring dimensions we must check the stress. The maximum allowable flexural operating stress for infinite fatigue life in these composite springs is 138 MPa for "spring configuration" and 100 MPa for "crossply configuration" .

The equation for the stress on a cantilever beam due to the force P_f is derived from :-

$$\alpha = \frac{Mc}{I} \text{ -----------------------------------(6)}$$

where M=moment= $P_f l$, c=distance t/2 and I= 1/12 bt^2

substitute for M, c and I and rearrange and we get :-

$$\alpha_f \equiv \frac{6P_f l}{bt^2} \text{ ---------------------------------(7)}$$

where b=width and t=thickness

This is the stress due to the bending action of the drive but there is also a stress due to the column load of the mass.

The bending stress due to the column load m is calculated using equation (6) but this time the moment M is calculated from M=P_d x 2Δ and hence the bending stress due to the column load is:-

$$\alpha_d \equiv \frac{12P_d\Delta}{bt^2} \text{ --------------------------------(8)}$$

Therefore the total bending stress on the spring is:-

$$\alpha \equiv \frac{6P_f l}{bt^2} + \frac{12P_d\Delta}{bt^2} \text{ ---------------------------------(9)}$$

Because the leaf springs are normally angled at between 20 and 25 degrees to the vertical the column load and second part of this equation is usually quite small and is often neglected.

If the stress level for the calculated spring size exceeds the maximum allowable stress, then the design must be changed. Luckily in practice this type of equipment reaches a large number of cycles very quickly and hence if we have a stress related problem it usually shows as a failure at a very early stage in the running life. (usually in the first week) If we get this then from equation (9) we can see that there are only 4 major ways that we can reduce this stress.

1. Reduce P, the force on the spring.

2. Reduce l, the spring length.

3. Increase b, the spring width.

4. Increase t, the spring thickness.

We are restrained by the changes that we can make as we must keep the same relative spring stiffness to keep the unit operating at natural frequency. If we find an overstress situation in a practical situation then we will also be restrained by any fixed design characteristics.

For these reasons the most common solution is to substitute two or more springs for the original one at each spring location. This is known as "banking" the springs and although it is possible to go through all the previous calculations again in order to calculate the thickness and stress there is a simple formula to design multiple springs with the same combined stiffness as the original single spring:-

$$n_1E_1t_1^3=n_2E_2t_2^3 \text{------------------------------} (10)$$

where n,E and t are the number, modulus and thickness respectively.

Chapter

5

Design Example 1

Let's take a look at a practical example.

How do we design springs for a new natural frequency conveyor? Let's assume that we have already calculated the tray size and desired stroke from the throughput requirements.

These are the design parameters that we need to know:-

1. Weight of the tray? This is the total weight supported by the springs.
2. Total weight of conveyed material in the tray?
3. Type of material? This is just to check if there are any special requirements of the material that we must consider. E.g. Damp sand will require different considerations to dry sand.
4. Number of spring "hangers" we think that we would like to use? (this may be changed later if the stress is too high)
5. Number of springs that we think that we will use on each hanger. **Tip if we have no idea then try 2 if there are plenty of hangers then 1 may be enough.**
6. Desired operating or drive frequency? (F)
7. Width of the springs? (b)
8. Total spring length?
9. Total unsupported (free) spring length when the spring is mounted on the hangers?
10. Desired machine stroke? (this is the total deflection 4Δ)

For Example 1 let's suppose that we have the following parameters:-

Weight of Tray	= 60 kg
Material Weight	= 5 kg
Material Type	= Dry cereal
Number of hangers	= 6
Number of springs per hanger	= 1
Desired operating frequency (F)	= 25 Hz
Spring Width (b)	= 38 mm
Total free length (2l)	= 100 mm
Total stroke (4Δ)	= 3 mm

- Determine mass on each spring (m)

$$m = \frac{Traywt + 20\%\, MaterialWt}{numberofsprings}$$

note that you use 20% of the material Weight as a realistic estimate of the mass of the conveyed material being supported by the springs at any instant while running. (20% assumes free flowing material) This means that the material weight is not very important.

m=(60+1)/6= 10.17 kg

- Determine the spring constant K from equation (3)

$$K=(25/5.03)^2 * 10.17 = 251 \text{ N/mm}$$

- Calculate the spring thickness from equation (5)

$$t^3 = \frac{4 * P * 50^3}{28 * 10^3 * 38 * 0.75}$$

and P=K*2Δ=251*1.5=376 and $E=28*10^3$

Therefore $t^3=236$ **t=6.18 mm**

6.50 mm is the nearest stock thickness so this size is tested in practice.

- Now we must check if the stress is OK by using equation (9)

$$\alpha = \frac{6*376*50}{38*6.18^2} = 78 \quad \text{MPa}$$

The maximum stress permitted is 138 and thus this set up is OK to try.

Design Example 2

We will not go through all the calculations again - you can practice on them yourselves - we will just give the parameters and the results.

For example 2 we have the following parameters in a bowl feeder :-

Bowl Wt 20kg feeding small plastic parts Wt 0.2kg using 3 hangers with 1 spring/hanger and an electromagnetic drive frequency 50 Hz. The spring free length is 75mm , the width is 25mm and the stroke required is 3mm.

If we calculate what thickness spring we require as above we get **t=7.37 mm.**

Then when we check the stress we get **α = 165 MPa**. As this is **above** the stress limit for infinite life of **138** we have to redesign as this setup would mean that although the springs should work they will be overstressed and we will expect to see problems within the first few days of operation.

Now if we do not wish to change any other of the parameters the easiest way to redesign is to change number of springs per hanger. We could go through the full calculation again but it is quicker to use equation (10) .

$$(7.37)^3 = 2 * t_2^3$$

$$t_2 = 5.85 \text{ mm}$$

I have not included the column load stress in any of the stress calculations in these examples

Now we check the stress again and we get α = 131 MPa which is OK to use so the result is that we should set up the bowl feeder with 2 springs/hanger . As the nearest thickness in practice are 5.00 and 6.50 mm we can try a setup with one of each of these thickness per hanger and fine tune accordingly .

Design Example 3

For example 3 we have a large rock screen in a quarry with the following parameters :-

Screen wt is 585 kg with rock wt of 460 kg in screen. There are 24 hangers with 1 spring/hanger being driven by a large out of balance motor at 12 Hz. The spring width is 100mm , the free length is 170mm and the stroke required is 20mm.

1. If we calculate the thickness required we get **t = 6.57 mm**

   Checking stress and we get **α = 190 MPa** - this is too high

2. Substituting 2 springs/hanger we get **t_2 = 5.20 mm**

   and recalculating the stress we get **α_2 = 151 MPa** - still too high

3. Substituting 3 springs/hanger we get **t_3= 4.55 mm**

   and recalculating stress we get **α_3 = 132 MPa - OK to try**

Thus we should set up this vibratory rock screen with 3 springs per hanger each 4.55 mm thick.

If you want us to work out some calculations for your equipment just fax us the equipment details on 01782-610033

Chapter

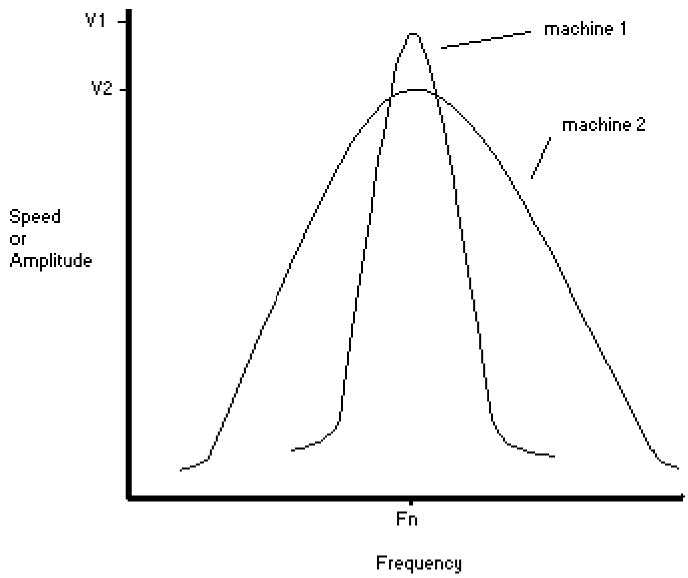
6

Practical Work

OK so what can we learn from existing equipment?

In fact with a minimum of equipment we can learn quite a lot. If we take some measurements of the throughput of our equipment at different frequencies we can plot the following curves:-

Fig 3



WHICH MACHINE IS THE BEST 1 OR 2 ?

?

In fact both machines can be equally suitable depending upon our requirements. Machine 1 has the highest maximum speed so if this is the main criteria then you can optimize the set up to produce this type of curve but be aware that because the curve is narrower and steeper this machine will be more susceptible to deviations such as the load which may give a more variable speed. Machine 2 does not have as high a maximum speed but it will be easier to fine tune and less prone to speed variations so if your requirement is for such things as a machine with multiple trays operating at the same speed then machine 2 will give the best results.

Appendix

a

Stock Sizes

THICKNESS			ORIENTATION	
mm	**inches**	**Ply**	**Spring**	**Crossply**
0.51	**0.020**	**2**		√
0.76	**0.030**	**3**		√
1.27	**0.050**	**5**		√
1.78	**0.070**	**7**		√
2.29	**0.090**	**9**		√
2.79	**0.110**	**11**		√
3.30	**0.130**	**13**	√	
3.81	**0.150**	**15**		√
4.06	**0.160**	**16**	√	
5.00	**0.195**	**19**	√	
6.50	**0.255**	**25**	√	
8.00	**0.315**	**31**	√	
9.53	**0.375**	**37**	√	
11.05	**0.435**	**43**	√	
12.95	**0.510**	**51**	√	
16.00	**0.630**	**63**	√	

Note that other thickness can be made available with production lead-times . Also check for minimum order quantities for non stock thickness.

Appendix b

Product Information*

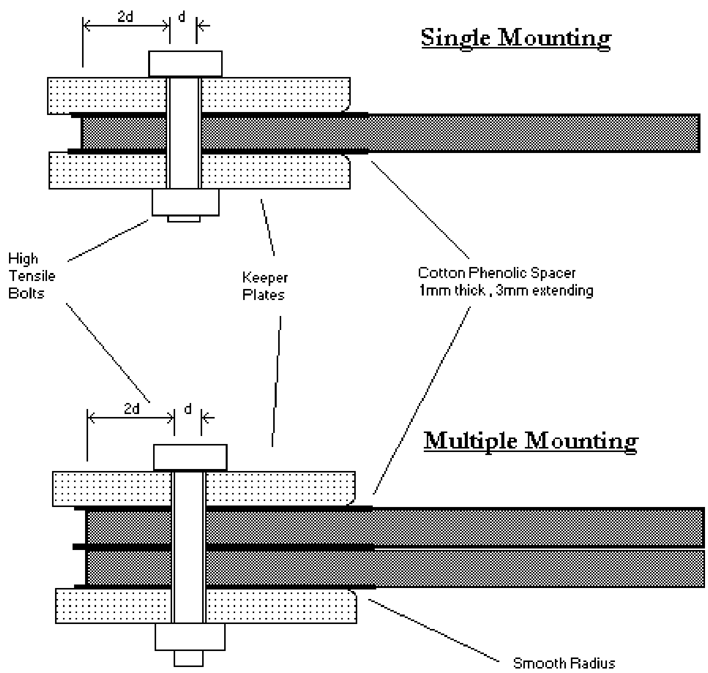
Heathcote Plastics Spring and Crossply Orientation composite leaf Springs.

Property	Spring	Crossply	Units
Flexural Strength	932	760	MPa
Flexural Modulus	28	22	GPa
Tensile Strength	800	480	MPa
Tensile Modulus	33	23	GPa
Compressive Strength at 0°	724	690	MPa
Compressive Strength at 90°	315	690	MPa
Short Beam Shear (5:1)		62	MPa
Maximum Stress for Infinite Fatigue life	138	100	MPa
Thermal Conductivity	0.34	0.34	W/M°K

*note that we have only included the main technical data required for spring design. If you require more data please call our Technical helpline on (+44) (0)1782 444219.

Mounting Principle

Fig 4



Good quality flat Keeper plates with a clean radius on the flexing edge are very important. If good quality keeper plates are ensured then it is possible to leave out the phenolic spacers which can be a source of dust.

The bolts should be torqued to the maximum permitted and then after running the equipment for a short time the bolts should be re - torqued.

Appendix d

To Heathcote Industrial Plastics

Quotation Request to *info@heathcotes.com* or
Fax No 01782-610033

Please Fax me a quote for the following composite leaf springs-

Please email /Fax the quote to –

Name	Company	Email/Fax address

The Quantity that we are interested in is…………………………….

The size that we require is as follows:-

Make a simple sketch of the spring with holes and hole centres or add a copy of your own drawing or email copy of file in DXF format